Introduction

Many test facilities run long term tests based on road-load simulations. These tests attempt to simulate actual load conditions, and therefore are not likely to utilize clean sinusoidal signals to drive the loading. There has been great interest expressed in incorporating Thermoelastic Stress Analysis (TSA) techniques along with these tests. It can be shown that the magnitude of thermal signals induced by load shapes from random or high frequency signals can be determined through a simple least squares approach.

Least Squares Method

Assume that a body has two independent load inputs, \( f \) and \( g \). The dynamic thermal signal will be a superposition of these two time histories over a period of time referred to as the accumulation window. To find the respective influences of two independent inputs a least-squares technique is employed. The fitting function takes the form of

\[
Y_n = A + Bf_n + Cg_n
\]  

(1)

where \( A \) is a DC offset \( B \) is the influence coefficient of the first reference, \( f_n \) is the first reference, \( C \) is the influence coefficient of the second reference, and \( g_n \) is the second reference. To calculate the influence coefficients the variance between the sum of the reference functions and the camera signal is minimized,

\[
\Delta^2 = \sum_{n=1}^{N} (y_n - Y_n)^2
\]  

(2)

The coefficients minimizing the variance are found by solving for the zeros of the partial derivatives with respect to the coefficients, \( A, B, \) and \( C \).

\[
\frac{\partial \Delta^2}{\partial A} = \frac{\partial}{\partial A} \sum_{n=1}^{N} (y_n - A - Bf_n - Cg_n)^2 = 0
\]

\[
\frac{\partial \Delta^2}{\partial B} = \frac{\partial}{\partial B} \sum_{n=1}^{N} (y_n - A - Bf_n - Cg_n)^2 = 0
\]  

(3)

\[
\frac{\partial \Delta^2}{\partial C} = \frac{\partial}{\partial C} \sum_{n=1}^{N} (y_n - A - Bf_n - Cg_n)^2 = 0
\]

This results in three equations and three unknowns. In matrix form this is stated

\[
\begin{bmatrix}
N & f_n & g_n & A \\
=f_n & f_n & B & y_n \\
g_n & g_n & C & y_n & g_n
\end{bmatrix}
\]

(4)

Single Low Frequency Operation

A single frequency load function can be modeled by setting the input functions to

\[
f_n = \sin \left( \frac{2\pi n}{N} \right) g_n = \cos \left( \frac{2\pi n}{N} \right)
\]  

(5)

Any amplitude and phase shift can be represented by the sum of these functions. By setting the accumulation period as an exact multiple of the load period,

\[
\sum f_n = 0, \sum g_n = 0, \sum f_n g_n = \sum g_n f_n = 0
\]  

(6)

the matrix diagonalizes down to

\[
\begin{bmatrix}
N & 0 & 0 & A \\
0 & \sum (f_n)^2 & 0 & \sum y_n \\
0 & 0 & \sum (g_n)^2 & C & \sum y_n g_n
\end{bmatrix}
\]

(7)

which can be easily solved so that,

\[
B = \frac{\sum y_n f_n}{\sum (f_n)^2}
\]

\[
C = \frac{\sum y_n g_n}{\sum (g_n)^2}
\]  

(8)

Notice that these relations are similar to those describing a lock-in algorithm, or a single term of a Fourier series. In fact, the operation of Eqs. 8 is identical to the function operated on by an analog lock-in system, where the signal is mixed with the reference function and integrated over time. Typically, the reference signals are cleaned-up and normalized in analog or digital electronics processing, so that the denominators in Eqs. 8 are constants and do not need to be calculated.

Single Random Input

In the case of only a single random input, the matrix of Eq. 4 can be partitioned and solved using Cramer’s rule, yielding
Null Reference

If the sampling rate is sufficiently close to a multiple of the loading frequency than a significant reduction in detected signal can result. A small perturbation from the frame rate results in a drastically different aliased signal. To avoid these “drop-out” frequencies the DeltaTherm system has been modified so that the operator can switch to an alternate frame rate with different “drop-out” frequencies. Figure 3 shows the response for a given signal amplitude over a range of frequencies. Two frequency choices are superimposed to show complete coverage of the frequency spectrum.

Integration Time

For high frequency a shortened sampling time within each frame is necessary. Integration times that are longer than the load period will suffer attenuation due to filtering effects. However, extremely short sample times do not take advantage of the available signal photons. It can be shown that a sample time equal to 37% of the load period optimizes the S/N ratio for BLIP detectors. If the camera is not background noise limited then the optimal operating integration time will be 50% of the load period.

High Frequency Examples

Figure 4 shows high stress regions on a turbine blade excited at 330 Hz, and figure 5 shows a beam in bending at high frequency excitation.
a) random signal image  

b) sinusoidal loading  

c) a random load lock-in mode  

Fig. 1 Random load performance comparison  

Fig. 2 Random load spectrum  

Fig. 3. Amplitude response incorporating frequency adjustment feature
Fig. 4  Turbine blade excited at 330 Hz

Fig. 5  Aluminum bar fixed at center and excited at 825 Hz