THERMOELASTIC MEASUREMENT UNDER RANDOM LOADING

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Introduction

Many test facilities run long term tests based on road-load simulations. These tests attempt to simulate actual load conditions, and therefore are not likely to utilize clean sinusoidal signals to drive the loading. There has been great interest expressed in incorporating Thermoelastic Stress Analysis (TSA) techniques along with these tests. It can be shown that the magnitude of thermal signals induced by load shapes from random or high frequency signals can be determined through a simple least squares approach.

Least Squares Method

Assume that a body has two independent load inputs, f and g. The dynamic thermal signal will be a superposition of these two time histories over a period of time referred to as the accumulation window. To find the respective influences of two independent inputs a least-squares technique is employed. The fitting function takes the form of

$$Y_n = A + Bf_n + Cg_n \qquad (1)$$

where A is a DC offset B is the influence coefficient of the first reference, f_n is the first reference, C is the influence coefficient of the second reference, and g_n is the second reference. To calculate the influence coefficients the variance between the sum of the reference functions and the camera signal is minimized,

$$\Delta^2 = \sum_{n=1}^{N} (y_n - Y_n)^2 \qquad (2)$$

The coefficients minimizing the variance are found by solving for the zeros of the partial derivatives with respect to the coefficients, A, B, and C.

$$\frac{\partial \Delta^2}{\partial A} = \frac{\partial}{\partial A}\sum_{n=1}^{N}\left(y_n - A - Bf_n - Cg_n\right)^2 = 0$$
$$\frac{\partial \Delta^2}{\partial B} = \frac{\partial}{\partial B}\sum_{n=1}^{N}\left(y_n - A - Bf_n - Cg_n\right)^2 = 0 \qquad (3)$$
$$\frac{\partial \Delta^2}{\partial C} = \frac{\partial}{\partial C}\sum_{n=1}^{N}\left(y_n - A - Bf_n - Cg_n\right)^2 = 0$$

This results in three equations and three unknowns. In matrix form this is stated

$$\begin{bmatrix} N & f_n & g_n \\ f_n & (f_n)^2 & f_n g_n \\ g_n & g_n f_n & (g_n)^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} y_n \\ y_n f_n \\ y_n g_n \end{bmatrix} \qquad (4)$$

Single Low Frequency Operation

A single frequency load function can be modeled by setting the input functions to

$$f_n = \sin\left(2\pi\frac{n}{N}\right) \quad g_n = \cos\left(2\pi\frac{n}{N}\right) \qquad (5)$$

Any amplitude and phase shift can be represented by the sum of these functions. By setting the accumulation period as an exact multiple of the load period,

$$\sum f_n = 0\,,\ \sum g_n = 0\,,\ \sum f_n g_n = \sum g_n f_n = 0 \qquad (6)$$

the matrix diagonalizes down to

$$\begin{bmatrix} N & 0 & 0 \\ 0 & \sum (f_n)^2 & 0 \\ 0 & 0 & \sum (g_n)^2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum y_n \\ \sum y_n f_n \\ \sum y_n g_n \end{bmatrix} \qquad (7)$$

which can be easily solved so that,

$$B = \frac{\sum y_n f_n}{\sum (f_n)^2} \qquad C = \frac{\sum y_n g_n}{\sum (g_n)^2} \qquad (8)$$

Notice that these relations are similar to those describing a lock-in algorithm, or a single term of a Fourier series. In fact, the operation of Eqs. 8 is identical to the function operated on by an analog lock-in system, where the signal is mixed with the reference function and integrated over time. Typically, the reference signals are cleaned-up and normalized in analog or digital electronics processing, so that the denominators in Eqs. 8 are constants and do not need to be calculated.

Single Random Input

In the case of only a single random input, the matrix of Eq. 4 can be partitioned and solved using Cramer's rule, yielding

$$B = \frac{\begin{vmatrix} N & \sum y_n \\ \sum f_n & \sum y_n f_n \end{vmatrix}}{\begin{vmatrix} N & \sum f_n \\ \sum f_n & \sum (f_n)^2 \end{vmatrix}} = \frac{N\sum y_n f_n - \sum y_n \sum f_n}{N\sum (f_n)^2 - \left(\sum f_n\right)^2} \qquad (9)$$

This algorithm will find the correlation between the dynamic thermal signal and the dynamic reference signal. Eqs.8 and Eq. 9 have been implemented in an instrument with multiple image processing channels: X, Y, and DC. All offsets, whether from the camera or the reference source are rejected. The individual sums seen in this relation are calculated in the electronics, but the final difference and ratio is calculated in the software. The summation of the left term in the numerator (Eq.9) is calculated as the X image. The row dependent sum of the squared reference signal needed to calculate the left summation in the denominator is stored in the right-most column of the X image. The Y image contains the summation of the signals found in the right term in the numerator. The right column of the Y image holds the summation of the reference signal found in the right term in the numerator and in the right in the denominator. The right column of the DC image holds the value of N. The right hand columns of all three images are eliminated from the data sets, because they store the information described above instead of actual signal data. Therefore, random images are 127x128 pixels.

Random Loading Examples

An acrylic sample was notched and then cyclically loaded to generate a stress concentration. A standard TSA image was taken using constant amplitude sinusoidal loading (Fig. 1b) to use as a comparison with the random images. Then a random load history (Fig. 2) was used to test the system's ability to detect and process a random TSA signal. The random image in Fig. 1a compares well with the sinusoidal image, and demonstrates the system's ability to process a wider variety of load signals. Previously, the system could only interpret sinusoidal type signals, and when a random loading signal was encountered very little signal would be detected. This is demonstrated in Fig. 1c where the acrylic specimen was loaded with a random signal and processed through the sinusoidal reference conditioners. Very little signal is detected.

High Frequency

Equation 8 can be also used for aliased high-frequency work. In this technique the frame rate of the camera is much less than that of the signal to be analyzed. If the sample time or integration time of the detectors is sufficiently short aliased wave forms can be sampled to solve for the magnitude of the high-frequency thermal signal. As in the sinusoidal case, it can be assumed that over long accumulation times the cross terms go to zero. In this case though, the reference signals are sampled and the denominators are calculated. A standard DeltaTherm 1000 system for TSA utilizes a 434 Hz frame rate camera which results in a practical frequency limit of about 200 Hz. The frequency range of the DeltaTherm 1000 system was extended from its current limit of 200 Hz to well over 2 kHz. The required modifications included the design and fabrication of a special signal reference card, electronic iris control, and the incorporation of additional signal processing circuits. The latter of these modifications was also required to adapt the DeltaTherm for the processing of variable amplitude (random) reference signals. The increased frequency range can require longer image acquisition times.

Null Reference

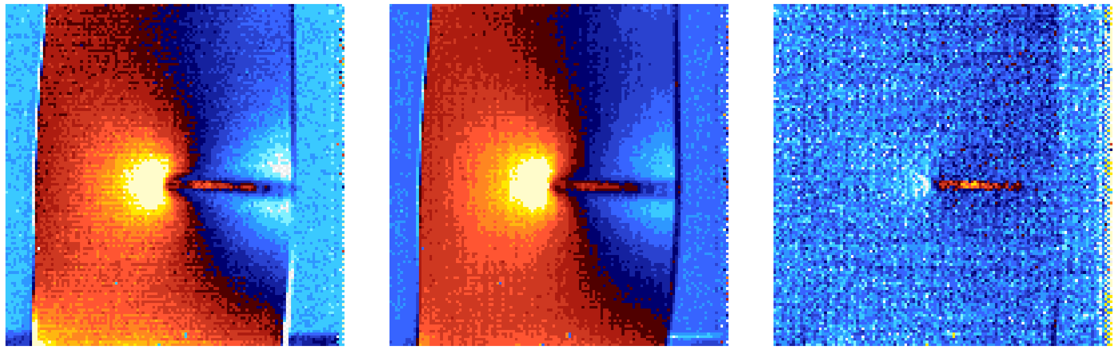
If the sampling rate is sufficiently close to a multiple of the loading frequency than a significant reduction in detected signal can result. A small perturbation from the frame rate results in a drastically different aliased signal. To avoid these "drop-out" frequencies the DeltaTherm system has been modified so that the operator can switch to an alternate frame rate with different "drop-out" frequencies. Figure 3 shows the response for a given signal amplitude over a range of frequencies. Two frequency choices are superimposed to show complete coverage of the frequency spectrum.

Integration Time

For high frequency a shortened sampling time within each frame is necessary. Integration times that are longer than the load period will suffer attenuation due to filtering effects. However, extremely short sample times do not take advantage of the available signal photons. It can be shown that a sample time equal to 37% of the load period optimizes the S/N ratio for BLIP detectors. If the camera is not background noise limited then the optimal operating integration time will be 50% of the load period.

High Frequency Examples

Figure 4 shows high stress regions on a turbine blade excited at 330 Hz, and figure 5 shows a beam in bending at high frequency excitation.



a) random signal image b) sinusoidal loading c) a random load lock-in mode

Fig. 1 Random load performance comparison

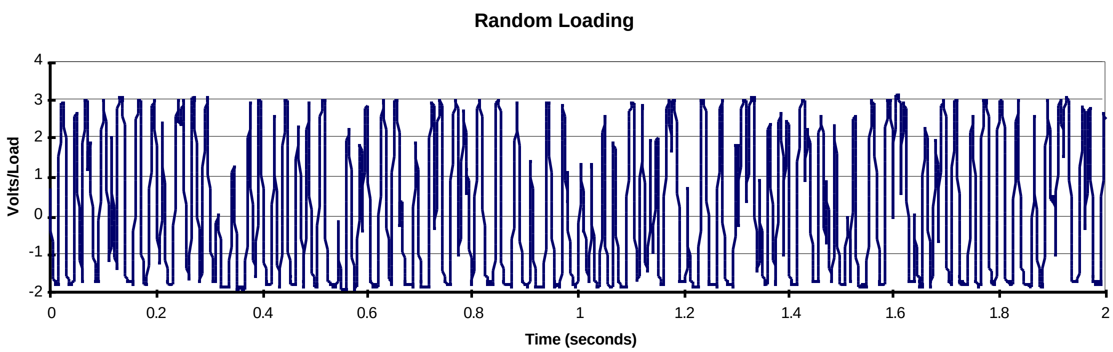


Fig. 2 Random load spectrum

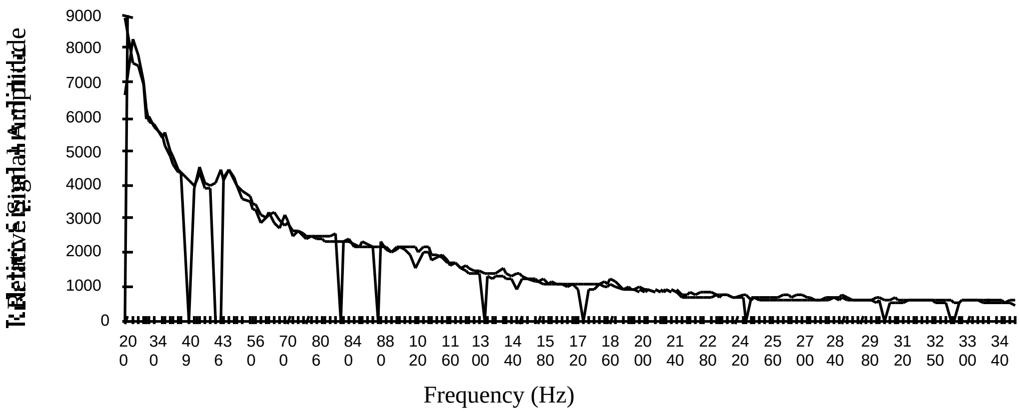


Fig. 3. Amplitude response incorporating frequency adjustment feature

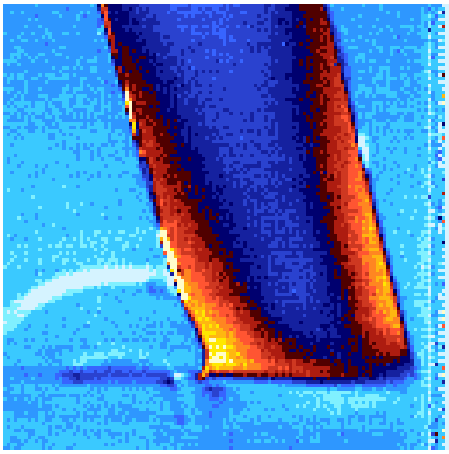


Fig. 4 Turbine blade excited at 330 Hz

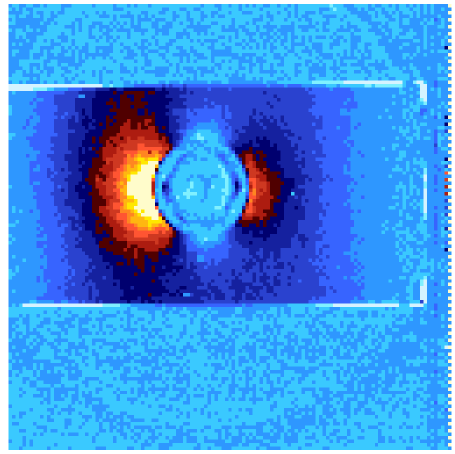


Fig. 5 Aluminum bar fixed at center and excited at 825 Hz